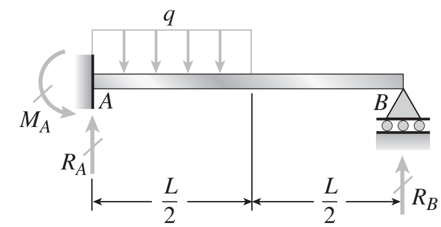
Problem 10.4-2 The propped cantilever beam shown in the figure supports a uniform load of intensity q on the left-hand half of the beam.

Find the reactions R_A, R_B, and M_A, and then draw the shear-force and bending-moment diagrams, labeling all critical ordinates.

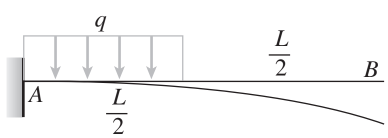


Solution 10.4-2 **Propped cantilever beam**

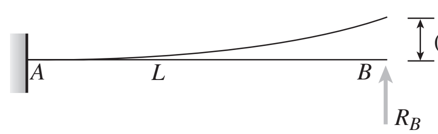
Select R_B as redundant.

EQUILIBRIUM $R_A = \frac{qL}{2} - R_B$ $M_A = \frac{qL^2}{8} - R_B L$

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$(\delta_B)_1 = \frac{7qL^4}{384EI}$



$(\delta_B)_2 = \frac{R_B L^3}{3EI}$

COMPATIBILITY $\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$

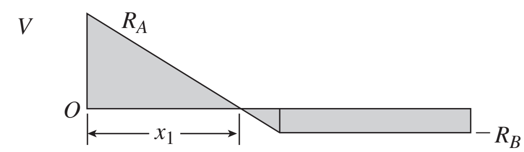
Substitute for $(\delta_B)_1$ and $(\delta_B)_2$ and solve for R_B:

$R_B = \frac{7qL}{128}$ ⟵

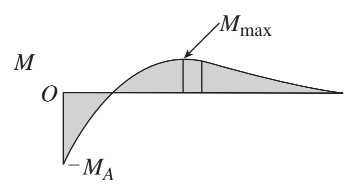
OTHER REACTIONS (FROM EQUILIBRIUM)

$R_A = \frac{57qL}{128}$ $M_A = \frac{9qL^2}{128}$ ⟵

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS



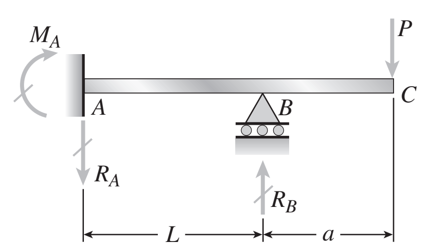
$x_1 = \frac{57L}{128}$



$M_{max} = \frac{945qL^2}{32,768}$

Problem 10.4-3 The figure shows a propped cantilever beam ABC having span length L and an overhang of length a. A concentrated load P acts at the end of the overhang.

Determine the reactions R_A, R_B, and M_A for this beam. Also, draw the shear-force and bending-moment diagrams, labeling all critical ordinates.



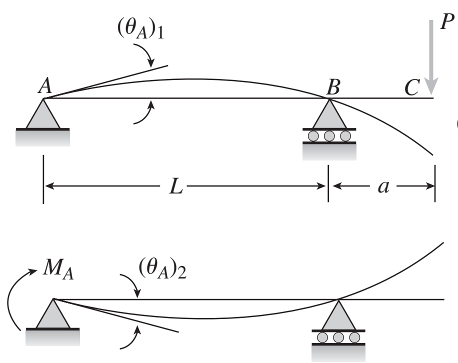
Solution 10.4-3 Beam with an overhang

Select M_A as redundant.

EQUILIBRIUM

$$R_A = \frac{1}{L}(M_A + Pa) \quad R_B = \frac{1}{L}(M_A + PL + Pa)$$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\theta_A)_1 = \frac{PaL}{6EI}$$

$$(\theta_A)_2 = \frac{M_A L}{3EI}$$

COMPATIBILITY $\theta_A = (\theta_A)_1 - (\theta_A)_2 = 0$

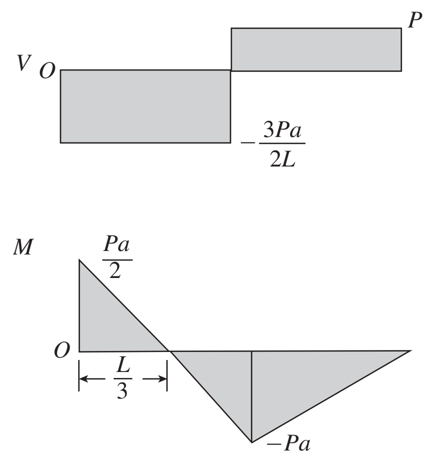
Substitute for $(\theta_A)_1$ and $(\theta_A)_2$ and solve for M_A:

$$M_A = \frac{Pa}{2} \quad \longleftarrow$$

OTHER REACTIONS (FROM EQUILIBRIUM)

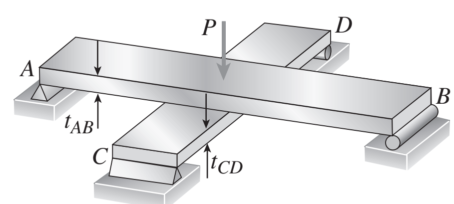
$$R_A = \frac{3Pa}{2L} \qquad R_B = \frac{P}{2L}(2L + 3a) \quad \longleftarrow$$

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS



Problem 10.4-4 Two flat beams AB and CD, lying in horizontal planes, cross at right angles and jointly support a vertical load P at their midpoints (see figure). Before the load P is applied, the beams just touch each other. Both beams are made of the same material and have the same widths. Also, the ends of both beams are simply supported. The lengths of beams AB and CD are L_{AB} and L_{CD}, respectively.

What should be the ratio t_{AB}/t_{CD} of the thicknesses of the beams if all four reactions are to be the same?



Solution 10.4-4 Two beams supporting a load P

For all four reactions to be the same, each beam must support one-half of the load P.

DEFLECTIONS

$$\delta_{AB} = \frac{(P/2)L_{AB}^3}{48EI_{AB}} \qquad \delta_{CD} = \frac{(P/2)L_{CD}^3}{48EI_{CD}}$$

COMPATIBILITY

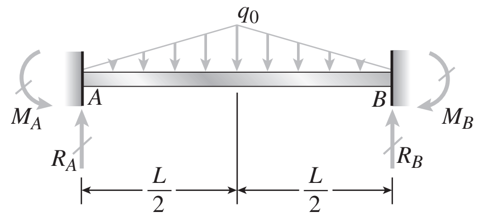
$$\delta_{AB} = \delta_{CD} \quad \text{or} \quad \frac{L_{AB}^3}{I_{AB}} = \frac{L_{CD}^3}{I_{CD}}$$

MOMENT OF INERTIA

$$I_{AB} = \frac{1}{12}bt_{AB}^3 \qquad I_{CD} = \frac{1}{12}bt_{CD}^3$$

$$\therefore \frac{L_{AB}^3}{t_{AB}^3} = \frac{L_{CD}^3}{t_{CD}^3} \qquad \frac{t_{AB}}{t_{CD}} = \frac{L_{AB}}{L_{CD}} \quad \longleftarrow$$

Problem 10.4-5 Determine the fixed-end moments (M_A and M_B) and fixed-end forces (R_A and R_B) for a beam of length L supporting a triangular load of maximum intensity q_0 (see figure). Then draw the shear-force and bending-moment diagrams, labeling all critical ordinates.



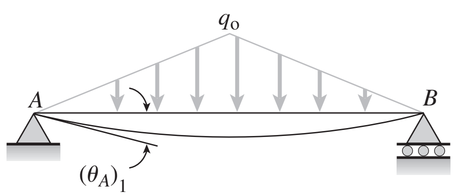
Solution 10.4-5 **Fixed-end beam (triangular load)**

Select M_A and M_B as redundants.

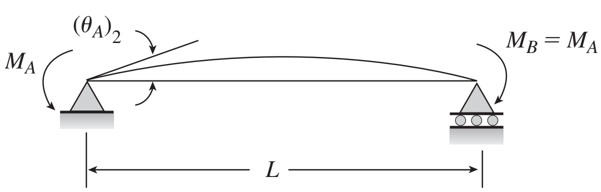
SYMMETRY $M_A = M_B \quad R_A = R_B$

EQUILIBRIUM $R_A = R_B = q_0L/4$ ←

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$$(\theta_A)_1 = \frac{5q_0L^3}{192EI}$$



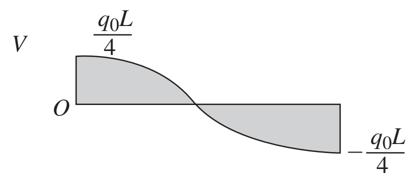
$$(\theta_A)_2 = \frac{M_AL}{2EI}$$

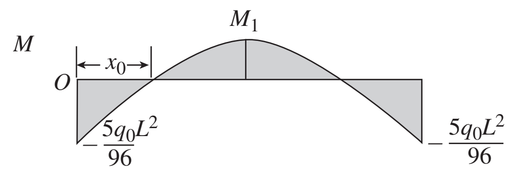
COMPATIBILITY $\theta_A = (\theta_A)_1 - (\theta_A)_2 = 0$

Substitute for $(\theta_A)_1$ and $(\theta_A)_2$ and solve for M_A:

$$M_A = M_B = \frac{5q_0L^2}{96} \quad \leftarrow$$

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS



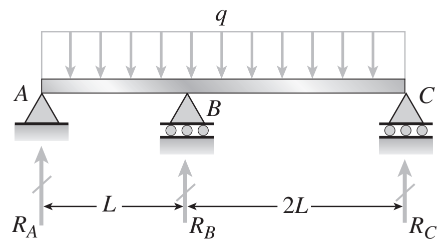


$$M_1 = \frac{q_0L^2}{32}$$

$$x_0 = 0.2231L$$

Problem 10.4-6 A continuous beam *ABC* with two unequal spans, one of length *L* and one of length 2*L*, supports a uniform load of intensity *q* (see figure).

Determine the reactions R_A, R_B, and R_C for this beam. Also, draw the shear-force and bending-moment diagrams, labeling all critical ordinates.



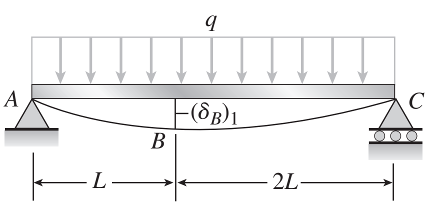
Solution 10.4-6 Continuous beam with two spans

Select R_B as redundant.

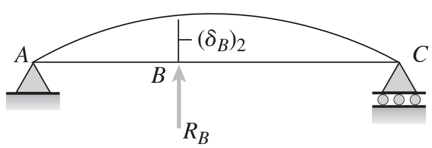
EQUILIBRIUM

$$R_A = \frac{3qL}{2} - \frac{2}{3}R_B \qquad R_C = \frac{3qL}{2} - \frac{1}{3}R_B$$

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



$$(\delta_B)_1 = \frac{11qL^4}{12\,EI}$$



$$(\delta_B)_2 = \frac{4R_BL^3}{9\,EI}$$

COMPATIBILITY

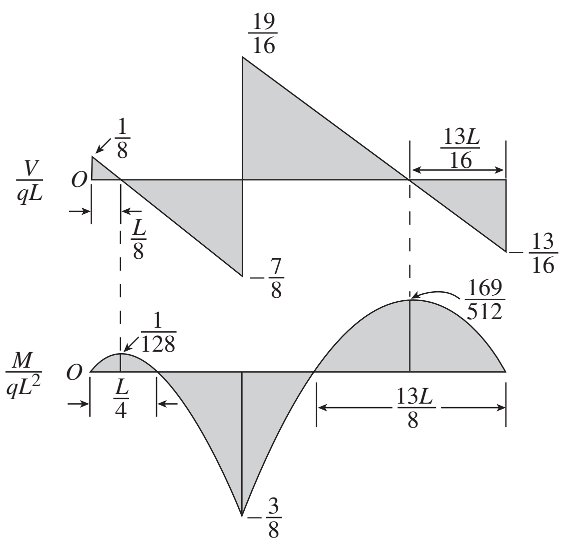
$$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$$

$$\frac{11qL^4}{12EI} - \frac{4R_BL^3}{9EI} = 0 \qquad R_B = \frac{33qL}{16} \quad \longleftarrow$$

OTHER REACTIONS (FROM EQUILIBRIUM)

$$R_A = \frac{qL}{8} \qquad R_C = \frac{13qL}{16} \quad \longleftarrow$$

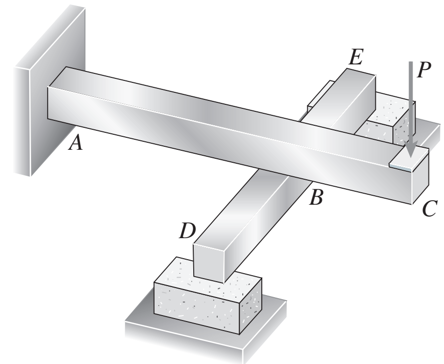
SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS

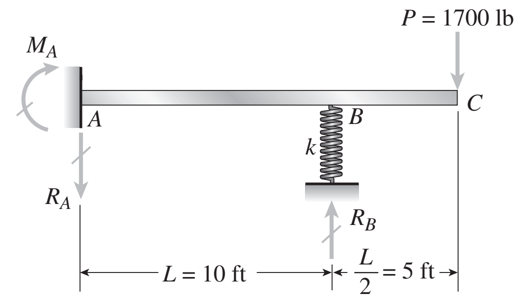


Problem 10.4-7 Beam *ABC* is fixed at support *A* and rests (at point *B*) upon the midpoint of beam *DE* (see the first part of the figure). Thus, beam *ABC* may be represented as a propped cantilever beam with an overhang *BC* and a linearly elastic support of stiffness *k* at point *B* (see the second part of the figure).

The distance from *A* to *B* is $L = 10$ ft, the distance from *B* to *C* is $L/2 = 5$ ft, and the length of beam *DE* is $L = 10$ ft. Both beams have the same flexural rigidity *EI*. A concentrated load $P = 1700$ lb acts at the free end of beam *ABC*.

Determine the reactions R_A, R_B, and M_A for beam *ABC*. Also, draw the shear-force and bending-moment diagrams for beam *ABC*, labeling all critical ordinates.





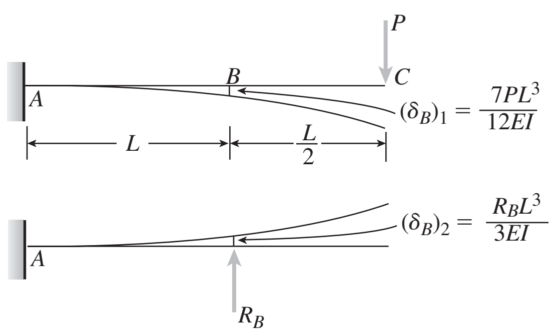
Solution 10.4-7 Beam with spring support

Select R_B as redundant.

EQUILIBRIUM

$R_A = R_B - P \qquad M_A = R_B L - 3PL/2$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



COMPATIBILITY $\delta_B = (\delta_B)_1 - (\delta_B)_2 = \dfrac{R_B}{k}$

Beam *DE*: $k = \dfrac{48\,EI}{L^3}$

$$\frac{7PL^3}{12\,EI} - \frac{R_B L^3}{3\,EI} = \frac{R_B L^3}{48EI} \qquad R_B = \frac{28P}{17} \quad \longleftarrow$$

OTHER REACTIONS (FROM EQUILIBRIUM)

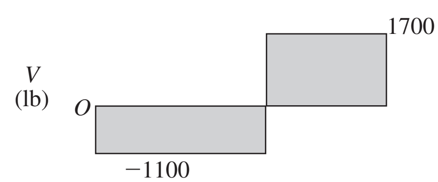
$$R_A = \frac{11P}{17} \qquad M_A = \frac{5PL}{34} \quad \longleftarrow$$

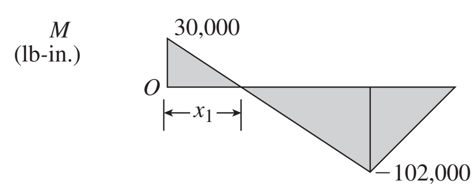
NUMERICAL VALUES

$P = 1700$ lb $\qquad L = 10$ ft $= 120$ in.

$R_A = 1100$ lb $\quad R_B = 2800$ lb
$M_A = 30{,}000$ lb-in. $\quad \longleftarrow$

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS

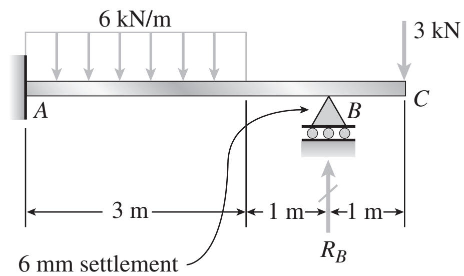




$$x_1 = \frac{300}{11}\text{ in.}$$
$$= 27.27\text{ in.}$$

Problem 10.4-8 The beam ABC shown in the figure has flexural rigidity $EI = 4.0$ MN·m^2. When the loads are applied to the beam, the support at B settles vertically downward through a distance of 6.0 mm.

Calculate the reaction R_B at support B.

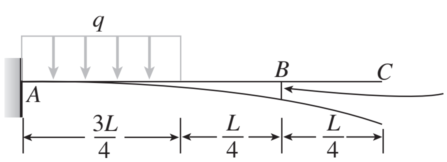


Solution 10.4-8 Overhanging beam with support settlement

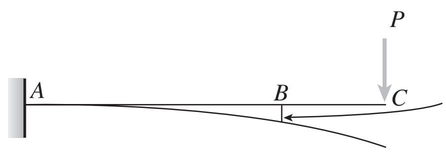
Select R_B as redundant.

Δ = settlement of support B

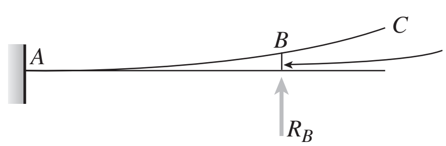
RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\delta_B)_1 = \frac{117qL^4}{2048EI}$$



$$(\delta_B)_2 = \frac{11PL^3}{24EI}$$



$$(\delta_B)_3 = \frac{R_B L^3}{3EI}$$

COMPATIBILITY $\delta_B = (\delta_B)_1 + (\delta_B)_2 - (\delta_B)_3 = \Delta$

Substitute for $(\delta_B)_1$, $(\delta_B)_2$, and $(\delta_B)_3$ and solve for R_B:

$$R_B = \frac{1}{2048}\left(351qL + 2816P - 6144\frac{EI\Delta}{L^3}\right) \longleftarrow$$

NUMERICAL VALUES

$q = 6.0$ kN/m $P = 3.0$ kN $\Delta = 6.0$ mm
$L = 4.0$ m $EI = 4.0$ MN · m^2

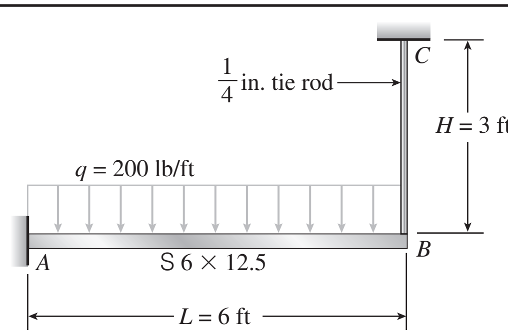
SUBSTITUTE INTO THE EQUATION FOR R_B

$R_B = 7.11$ kN $\longleftarrow$

Problem 10.4-9 A beam AB is cantilevered from a wall at one end and held by a tie rod at the other end (see figure). The beam is an S 6 × 12.5 I-beam section with length $L = 6$ ft. The tie rod has a diameter of 1/4 inch and length $H = 3$ ft. Both members are made of steel with $E = 30 \times 10^6$ psi. A uniform load of intensity $q = 200$ lb/ft acts along the length of the beam. Before the load q is applied, the tie rod just meets the end of the cable.

(a) Determine the tensile force T in the tie rod due to the uniform load q.

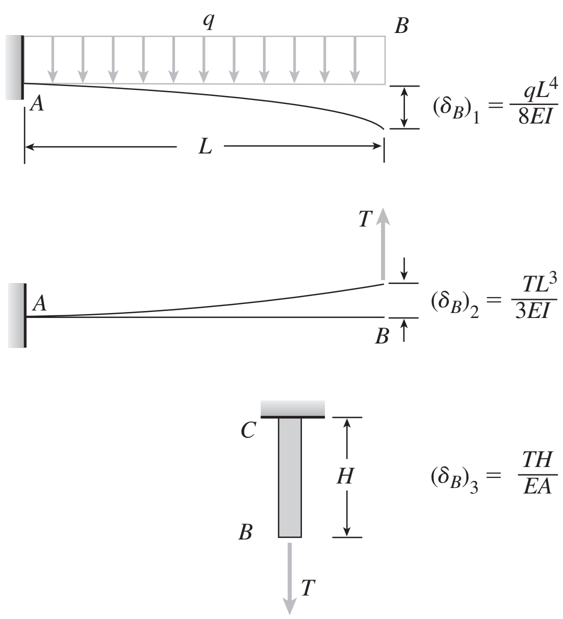
(b) Draw the shear-force and bending-moment diagrams for the beam, labeling all critical ordinates.



Solution 10.4-9 Beam supported by a tie rod

Select the force T in the tie rod as redundant.

RELEASED STRUCTURE AND FORCE-DISPLACEMENT RELATIONS



COMPATIBILITY $(\delta_B)_1 - (\delta_B)_2 = (\delta_B)_3$

or $$\frac{qL^4}{8EI} - \frac{TL^3}{3EI} = \frac{TH}{EA}$$

$$T = \frac{3qAL^4}{8AL^3 + 24IH} \quad \longleftarrow$$

NUMERICAL VALUES

$q = 200$ lb/ft $L = 6$ ft $H = 3$ ft

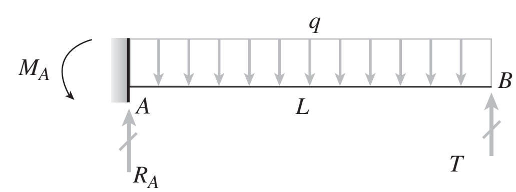
$E = 30 \times 10^6$ psi

Beam: S 6 × 12.5 $I = 22.1$ in.^{4}

Tie Rod: $d = 0.25$ in. $A = 0.04909$ in.^{2}

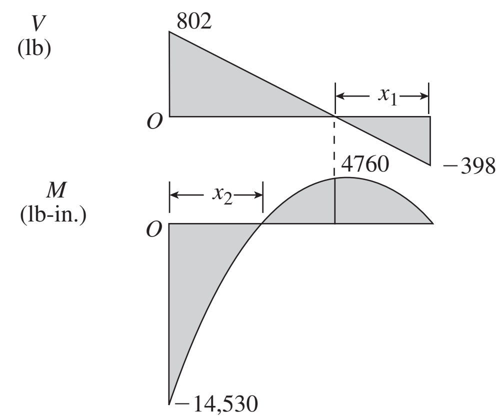
Substitute: $T = 398$ lb $\longleftarrow$

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS



$R_A = qL - T = 802$ lb

$M_A = \frac{qL^2}{2} - TL = 14{,}530$ lb-in.

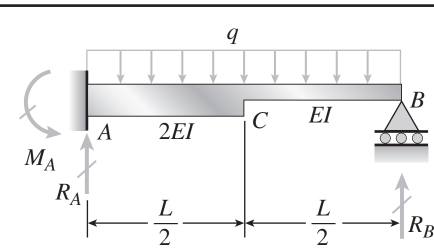


$x_1 = 23.9$ in.

$x_2 = 24.2$ in.

Problem 10.4-10 The figure shows a nonprismatic, propped cantilever beam AB with flexural rigidity $2EI$ from A to C and EI from C to B.

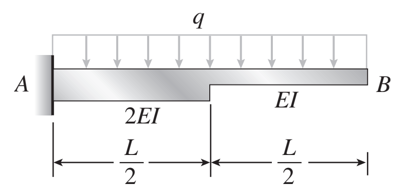
Determine all reactions of the beam due to the uniform load of intensity q. (*Hint:* Use the results of Problems 9.7-1 and 9.7-2.)



Solution 10.4-10 Nonprismatic beam

Select R_B as redundant.

RELEASED STRUCTURE



$(\delta_B)_1$ = downward deflection of end B due to load q

$(\delta_B)_2$ = upward deflection due to reaction R_B

FORCE-DISPLACEMENT RELATIONS

From Prob. 9.7-2: $\delta_B = \dfrac{qL^4}{128EI_1}\left(1 + 15\dfrac{I_1}{I_2}\right)$

$I_1 \rightarrow I \quad I_2 \rightarrow 2I \qquad \therefore (\delta_B)_1 = \dfrac{17\,qL^4}{256\,EI}$

From Prob. 9.7-1:

$\delta_B = \dfrac{PL^3}{24\,EI_1}\left(1 + 7\dfrac{I_1}{I_2}\right) \qquad \therefore (\delta_B)_2 = \dfrac{3R_BL^3}{16\,EI}$

COMPATIBILITY

$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0$

or

$$\frac{17qL^4}{256EI} - \frac{3R_BL^3}{16EI} = 0 \quad R_B = \frac{17qL}{48} \quad \longleftarrow$$

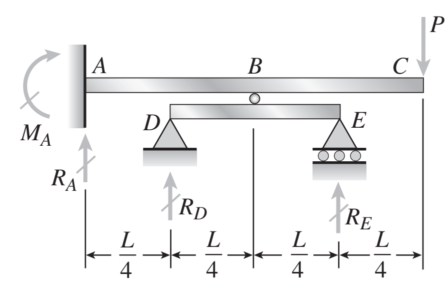
EQUILIBRIUM

$$R_A = qL - R_B = \frac{31qL}{48} \quad M_A = \frac{qL^2}{2} - R_BL = \frac{7qL^2}{48} \quad \longleftarrow$$

Problem 10.4-11 A beam ABC is fixed at end A and supported by beam DE at point B (see figure). Both beams have the same cross section and are made of the same material.

(a) Determine all reactions due to the load P.

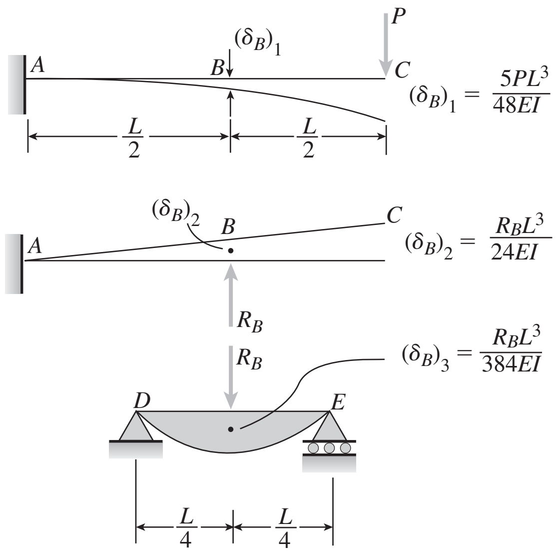
(b) What is the numerically largest bending moment in either beam?



Solution 10.4-11 Beam supported by a beam

Let R_B = interaction force between beams
Select R_B as redundant.

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



COMPATIBILITY $(\delta_B)_1 - (\delta_B)_2 = (\delta_B)_3$

Substitute and solve: $R_B = \frac{40P}{17}$ ⟵

SYMMETRY AND EQUILIBRIUM

$R_D = R_E = \frac{R_B}{2} = \frac{20P}{17}$ ⟵

$R_A = P - R_D - R_E = -\frac{23P}{17}$ ⟵

(minus means downward)

$M_A = R_B\left(\frac{L}{2}\right) - PL = \frac{3PL}{17}$ ⟵

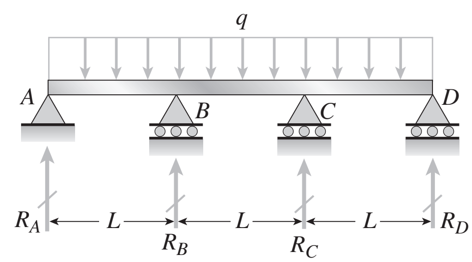
BEAM ABC: $M_{max} = M_B = -\frac{PL}{2}$

BEAM DE: $M_{max} = M_B = \frac{5PL}{17}$

$|M_{max}| = \frac{PL}{2}$ ⟵

Problem 10.4-12 A three-span continuous beam $ABCD$ with three equal spans supports a uniform load of intensity q (see figure).

Determine all reactions of this beam and draw the shear-force and bending-moment diagrams, labeling all critical ordinates.



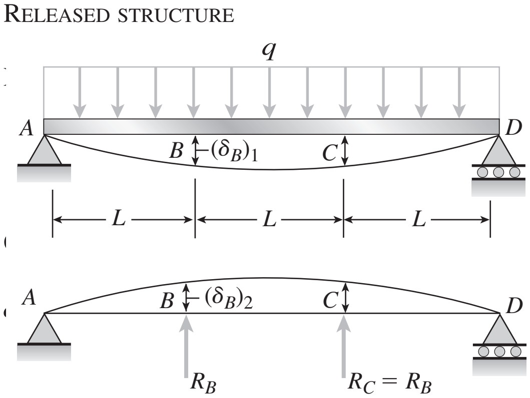
Solution 10.4-12 Three-span continuous beam

SELECT R_B AND R_C AS REDUNDANTS.

SYMMETRY AND EQUILIBRIUM

$R_C = R_B \quad R_A = R_D = \frac{3qL}{2} - R_B$

RELEASED STRUCTURE



FORCE-DISPLACEMENT RELATIONS

$(\delta_B)_1 = \frac{11qL^4}{12EI} \quad (\delta_B)_2 = \frac{5R_BL^3}{6EI}$

COMPATIBILITY

$\delta_B = (\delta_B)_1 - (\delta_B)_2 = 0 \quad \therefore R_B = \frac{11qL}{10}$ ⟵

OTHER REACTIONS

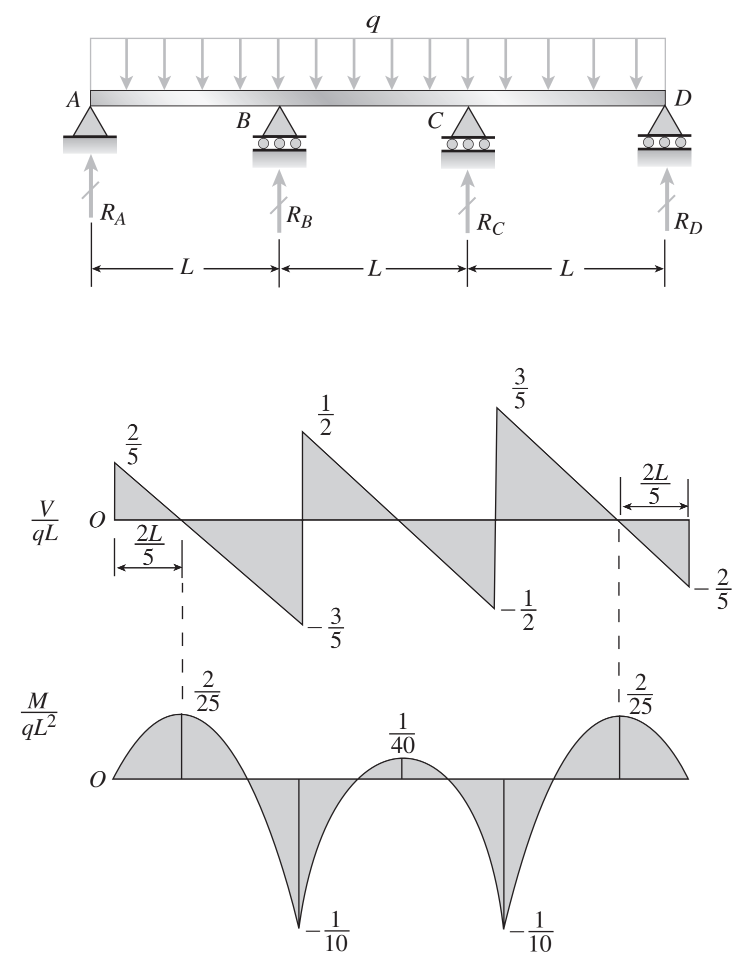
From symmetry and equilibrium:

$R_C = R_B = \frac{11qL}{10}$ ⟵

$R_A = R_D = \frac{2qL}{5}$ ⟵

(Continued)

LOADING, SHEAR-FORCE, AND BENDING-MOMENT DIAGRAMS



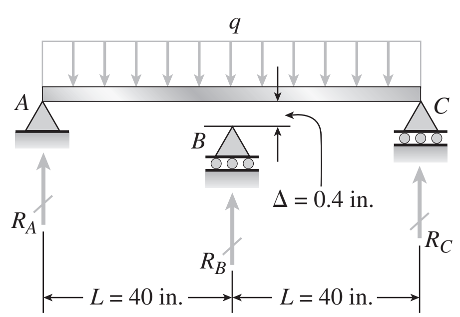
$$M_B = M_C = -\frac{qL^2}{10}$$

$$M_{max} = \frac{2\,qL^2}{25}$$

Problem 10.4-13 A beam *AC* rests on simple supports at points *A* and *C* (see figure). A small gap $\Delta = 0.4$ in. exists between the unloaded beam and a support at point *B*, which is midway between the ends of the beam. The beam has total length $2L = 80$ in. and flexural rigidity $EI = 0.4 \times 10^9$ lb-in.^{2}

Plot a graph of the bending moment M_B at the midpoint of the beam as a function of the intensity *q* of the uniform load.

Hints: Begin by determining the intensity q_0 of the load that will just close the gap. Then determine the corresponding bending moment $(M_B)_0$. Next, determine the bending moment M_B (in terms of *q*) for the case where $q < q_0$. Finally, make a statically indeterminate analysis and determine the moment M_B (in terms of *q*) for the case where $q > q_0$. Plot M_B (units of lb-in.) versus *q* (units of lb/in.) with *q* varying from 0 to 2500 lb/in.



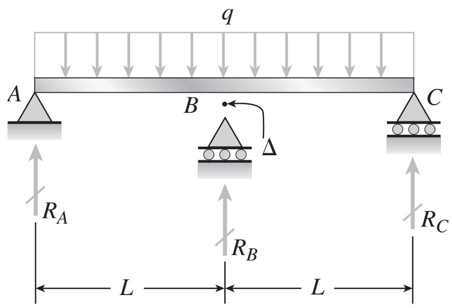
Solution 10.4-13 Beam on a support with a gap

q_0 = load required to close the gap

Δ = magnitude of gap

$(M_B)_0$ = bending moment when $q = q_0$

CASE 1 $\quad q < q_0$



$$\delta_B = \frac{5qL^4}{24EI}$$

$$M_B = \frac{qL^2}{2}$$

$R_A = R_C = qL$

CASE 2 $\quad q = q_0$

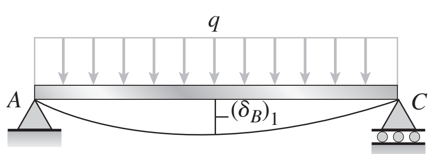
$$\delta_B = \Delta = \frac{5q_0L^4}{24EI} \quad q_0 = \frac{24EI\Delta}{5L^4} \qquad (1)$$

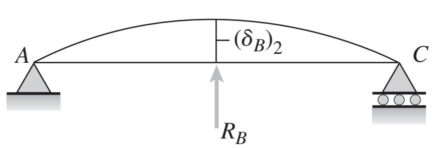
$$(M_B)_0 = \frac{q_0L^2}{2} = \frac{12EI\Delta}{5L^2} \qquad (2)$$

CASE 3 $\quad q > q_0$ (statically indeterminate)

Select R_B as redundant.

RELEASED STRUCTURE





$$(\delta_B)_1 = \frac{5qL^4}{24EI}$$

$$(\delta_B)_2 = \frac{R_BL^3}{6EI}$$

COMPATIBILITY $\quad \delta_B = (\delta_B)_1 - (\delta_B)_2 = \Delta$

$$\text{or} \quad \frac{5qL^4}{24EI} - \frac{R_BL^3}{6EI} = \Delta \quad R_B = \frac{5qL}{4} - \frac{6EI\Delta}{L^3}$$

EQUILIBRIUM

$R_A = R_C \quad 2R_A - 2qL + R_B = 0$

$$R_A = R_C = \frac{3qL}{8} + \frac{3EI\Delta}{L^3}$$

$$M_B = R_AL - \frac{qL^2}{2} = \frac{3EI\Delta}{L^2} - \frac{qL^2}{8}$$

NUMERICAL VALUES

$\Delta = 0.4$ in. $\quad L = 40$ in. $\quad EI = 0.4 \times 10^9$ lb-in.^{2}

Units: lb, in.

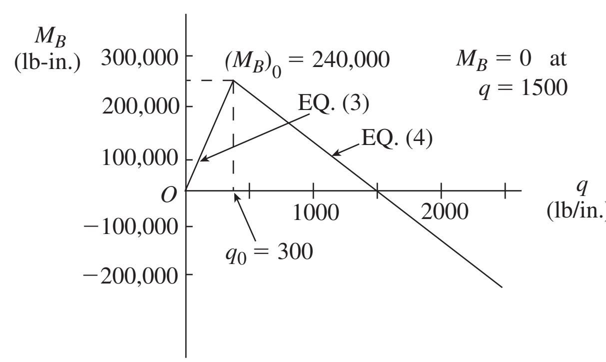
From Eqs. (1) and (2): $\quad q_0 = 300$ lb/in.

$(M_B)_0 = 240{,}000$ lb-in.

For $q < q_0$: $M_B = 800\,q \qquad (3)$

For $q > q_0$: $M_B = 300{,}000 - 200\,q \qquad (4)$

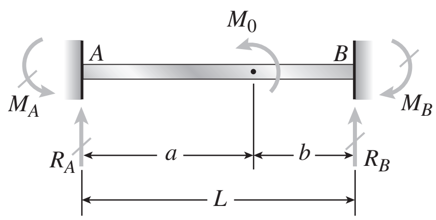
GRAPH OF BENDING MOMENT M_B (EQS. 3 AND 4)



Problem 10.4-14 A fixed-end beam AB of length L is subjected to a moment M_0 acting at the position shown in the figure.

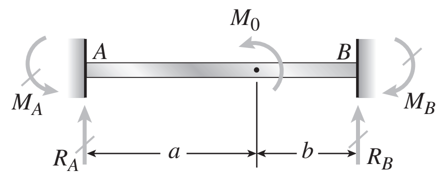
(a) Determine all reactions for this beam.

(b) Draw shear-force and bending-moment diagrams for the special case in which $a = b = L/2$.



Solution 10.4-14 **Fixed-end beam (M_0 = applied load)**

Select R_B and M_B as redundants.

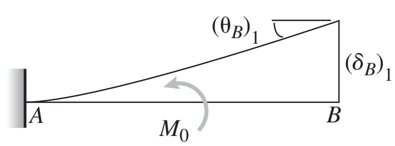


$L = a + b$

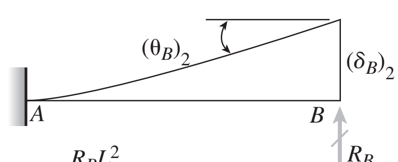
EQUILIBRIUM

$R_A = -R_B \qquad M_A = M_B - R_B L - M_0$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.

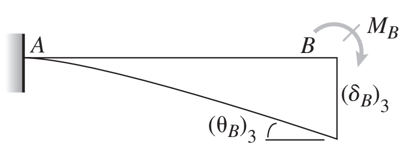


$(\theta_B)_1 = \dfrac{M_0 a}{EI} \qquad (\delta_B)_1 = \dfrac{M_0 a}{2EI}(a + 2b)$



$(\theta_B)_2 = \dfrac{R_B L^2}{2EI}$

$(\delta_B)_2 = \dfrac{R_B L^3}{3EI}$



$(\theta_B)_3 = \dfrac{M_B L}{EI} \qquad (\delta_B)_3 = \dfrac{M_B L^2}{2EI}$

$$(\theta_B)_1 = \frac{M_0 a}{EI} \qquad (\delta_B)_1 = \frac{M_0 a}{2EI}(a + 2b)$$

$$(\theta_B)_2 = \frac{R_B L^2}{2EI} \qquad (\delta_B)_2 = \frac{R_B L^3}{3EI}$$

$$(\theta_B)_3 = \frac{M_B L}{EI} \qquad (\delta_B)_3 = \frac{M_B L^2}{2EI}$$

COMPATIBILITY

$$\delta_B = -(\delta_B)_1 - (\delta_B)_2 + (\delta_B)_3 = 0$$

or $\quad 2R_B L^3 - 3M_B L^2 = -3M_0 a(a + 2b) \qquad (1)$

$$\theta_B = (\theta_B)_1 + (\theta_B)_2 - (\theta_B)_3 = 0$$

or $\quad R_B L^2 - 2M_B L = -2M_0 a \qquad (2)$

SOLVE EQS. (1) AND (2):

$$R_B = -\frac{6M_0 ab}{L^3} \qquad M_B = -\frac{M_0 a}{L^2}(3b - L) \quad \longleftarrow$$

FROM EQUILIBRIUM:

$$R_A = \frac{6M_0 ab}{L^3} \qquad M_A = \frac{M_0 b}{L^2}(3a - L) \quad \longleftarrow$$

SPECIAL CASE $a = b = L/2$

$$R_A = -R_B = \frac{3M_0}{2L} \qquad M_A = -M_B = \frac{M_0}{4}$$